Reciprocal dumping with Bertrand competition*

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Abstract

This paper examines if international trade can reduce total welfare in an international oligopoly with differentiated goods. We show that welfare is a U-shaped function in the transport cost as long as trade occurs in equilibrium. With a Cournot duopoly trade can reduce welfare compared to autarchy for any degree of product differentiation. Under Bertrand competition we show that trade may reduce welfare compared to autarchy, if firms produce sufficiently close substitutes and the autarchy equilibrium is sufficiently competitive. Otherwise it can not.

Keywords: Reciprocal dumping, intra-industry trade, oligopoly, product differentiation, transport costs


1 Introduction

The reciprocal dumping model (Brander, 1981, Brander and Krugman, 1983) shows that competition in quantities à la Cournot in segmented markets can generate two-way trade in the same good ("reciprocal dumping"), even though foreign and domestic goods are perfect substitutes. A striking conclusion from

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these models is that trade can reduce welfare. For high enough transport costs, the waste of shipping identical goods in both directions dominates the positive, pro-competitive, effect of trade.

The purpose of this paper is to analyze if this result extends to an international oligopoly in which firms produce differentiated products and compete in prices à la Bertrand. The question is particularly interesting since we for empirical purposes often think of firms as setting prices rather than quantities.\(^1\) It is also clear that it is rare that goods produced by different firms (let alone in different countries) are literally homogenous.\(^2\) We ask the following question: Can international trade reduce total welfare in an international oligopoly with differentiated goods and price competition? To provide a benchmark for this question we also provide an analysis of product differentiation when there is Cournot competition.

To the best of our knowledge Clarke and Collie (2003) is the only previous paper to examine the welfare effects of trade in an international oligopoly with differentiated products and price competition.\(^3\) They assume that there are two countries with one firm in each country. Utility increases in the number of products. The utility function they use was originally introduced by Bowley, 1924 (see Martin, 2002, for a discussion). Based on these assumptions they establish that trade always increases welfare compared to autarchy. Thus, the welfare result in the Brander-Krugman model, that trade can reduce welfare compared to autarchy, does not hold for a Bertrand duopoly with differentiated goods.

The result in Clark and Collie (2003) is, however, specific for duopoly. As our analysis shows, it does not generalize to other market structures. Allowing for more than one firm in each country, trade can reduce welfare compared to autarchy in a Bertrand oligopoly as long as there is sufficient competition between domestic producers and goods are sufficiently close substitutes. Otherwise it can not.

Let us give the reader some intuition for why the effects of trade may or may not generalize from the Brander-Krugman setup. The welfare result presented by Brander and Krugman has two properties.

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\(^1\)The rapidly expanding literature on competition with differentiated goods almost exclusively examines price as the strategic variable (see for instance Goldberg, 1995 or Berry, Levinsohn and Pakes, 1999). Indeed, in Friberg and Ganslandt (2005) we attempt to evaluate the welfare effects of trade in bottled water using Bertrand competition in differentiated goods.

\(^2\)The relation between the gains from trade and access to differentiated varieties has been the focus of much recent work, see for instance Broda and Weinstein (2004) for an empirical analysis or Romer (1994) for a theoretical discussion. Indeed already Brander (1981) concluded by noting that it would be interesting to study two-way trade in similar goods, and motivated the homogenous goods assumption as a convenient first step in analyzing the broader set of issues.\(^3\) It is clear that if firms compete in prices and goods are perfect substitutes we will not have reciprocal dumping. Ben-Zvi and Helpman (1992) show that two-way trade in homogenous goods also fails to emerge if we assume that firms in a first stage determine capacities and then in a second stage set prices.

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First, welfare falls when the trade cost is reduced from the prohibitive level as costly imported goods replace local production. Second, welfare may be higher in the autarchy equilibrium than in a more competitive trade equilibrium. The former result is a local property while the latter is global.

Two factors suggest that the result in Brander-Krugman does not necessarily extend beyond quantity competition and product homogeneity. The first significant factor is that price competition is more aggressive than quantity competition. The disciplining effect of imports on a sole domestic producer is stronger and less resources are wasted in trade to achieve a given pro-competitive effect. The second significant factor is that product differentiation - i.e. variety - has a direct positive effect on welfare since consumer value access to a greater variety of brands.

Other factors, on the other hand, suggest that trade may reduce welfare also under price competition. The most important is that entry in a market with differentiated products and price competition result in substitution (as long as the products are not completely independent). The entrant gains some market share while incumbents loose. Inefficient importation may, consequently, replace less costly local production. At least in principle, this opens for the possibility of an ambiguous net effect of international trade.

In this paper we shall therefore analyze how the welfare effects of competition, trade and love-of-variety interact. Section 2 first presents the duopoly results of Clarke and Collie as a benchmark and goes on to analyze the difference between a Cournot and a Bertrand duopoly. The third section introduces an additional firm in each country. Section four concludes.

2 Duopoly

2.1 Bertrand

Consider a model with two firms and two identical markets (home, denoted H, and foreign, denoted F). Each firm produces a symmetric but differentiated variety of a product demanded in both markets. In autarchy each firm has a monopoly in its domestic market and does not export to the other market.\footnote{This is a convenient starting point since the autarchy equilibrium does not depend on the assumption about Bertrand or Cournot behavior. The welfare level is consequently the same under autarchy in both models and the difference between Bertrand and Cournot competition depends only on the properties of the trade equilibrium.}
Markets are segmented. For simplicity focus on the equilibrium in the home market. Firms compete in a one-shot Bertrand fashion. The foreign firm must incur a trade cost, $t$, per unit transported from the foreign market to the home market and vice versa. The marginal cost of production is $c$ for each firm.

A representative consumer in the home market has the following utility function

$$U(q_H,q_F) = a(q_H + q_F) - \frac{1}{2}b(q_H^2 + 2\theta q_H q_F + q_F^2) + m$$

(1)

where $q_H$ is the quantity of firm H's variety and $q_F$ is the quantity of firm F’s variety and $m$ is the utility of money. The linear demand functions that result from utility maximization subject to the budget constraint are given in Appendix A. The profit in the home market for the home and foreign firm is, respectively:

$$\pi_H = \left( \frac{a(1 - \theta)}{b(1 - \theta^2)} - \frac{1}{b(1 - \theta^2)}p_H + \frac{\theta}{b(1 - \theta^2)}p_F \right) (p_H - c),$$

(2)

$$\pi_F = \left( \frac{a(1 - \theta)}{b(1 - \theta^2)} + \frac{\theta}{b(1 - \theta^2)}p_H - \frac{1}{b(1 - \theta^2)}p_F \right) (p_F - c - t),$$

(3)

with the corresponding first order conditions (reaction functions) for the home (Eq 4) and foreign firms (Eq 5):

$$p_H (p_F) = \frac{1}{2}a + \frac{1}{2}c - \frac{1}{2} \theta a + \frac{1}{2} \theta p_F,$$

(4)

$$p_F (p_H) = \frac{1}{2}a + \frac{1}{2}c - \frac{1}{2} \theta a + \frac{1}{2} \theta p_H + \frac{1}{2}t.$$  

(5)

The reaction functions are illustrated in Figure 1. The unit trade cost shifts the reaction curve for the foreign firm to the right, i.e. the trade cost has a positive effect on the foreign firm’s price. The trade cost is non prohibitive as long as the import volume is strictly positive in equilibrium. The set of possible equilibria with a strictly positive import volume is the line between point F ("free trade") and point B in the figure. Prices for which imports are exactly zero is illustrated with $q_F = 0$. For trade costs
above \( \hat{t} \) but below \( \bar{t} \) price is thus given by the bold line connecting points B ("binding constraint") and A ("autarchy"), since, if there is no entry restriction, the home firm would set price so as just to keep the foreign firm’s quantity at zero. There is, consequently, no trade but the potential for imports still restricts prices in the domestic market and there is a flavor of limit pricing to these equilibria. If trade costs are higher still (above \( \bar{t} \)) foreign entry is blocked and the domestic price is unconstrained and equal to the monopoly price \( p_H^* \).

Figure 1 about here

Alternatively, we could model a sequential game where the foreign firm first takes a decision to enter or stay out of the home market and then, subject to entry, sets a profit maximizing price in a non-cooperative fashion. The foreign firm enters market \( H \) if the expected profit in the Nash equilibrium is non-negative. Accordingly, in the sequential game the foreign firm enters if \( t \leq \hat{t} \) and stays out if \( t > \hat{t} \). With a sequential game we would thus see the domestic price jump from the point B to the unconstrained monopoly price as trade costs increase above \( \hat{t} \).

Define welfare as the sum of utility and foreign and domestic profits:

\[
W_B = U(q_H^B, q_F^B) + \pi_H (p_H^B, p_F^B) + \pi_F (p_H^B, p_F^B).
\]

We can then compare the welfare at different levels of transport costs.

**Proposition 1** (Clarke and Collie, 2003). Let utility be given by Eq (1) with \( \theta \in (0, 1) \). Let firms have constant marginal costs and per unit transport cost. Assume that there is one firm from each country and that these firms compete à la Bertrand. Then i) a small decrease in trade costs from the prohibitive level \( (\hat{t}) \) reduces welfare, ii) welfare is a U-shaped function of trade costs as long as trade occurs in equilibrium and iii) welfare under trade is higher than welfare under autarchy.

**Proof.** See appendix A for the relevant welfare expressions. To show i) we differentiate the equilibrium welfare level with respect to the trade cost and evaluate the derivative at the prohibitive level

\[
\left. \frac{dW}{dt} \right|_{t=\hat{t}+} = \frac{(a-c) \theta}{(2-\theta^2)(2-\theta)(2+\theta)b} > 0
\]
which shows that raising the trade cost, $t$, close to the prohibitive level increases total welfare. To show ii) note first that welfare is higher with free trade than at the prohibitive trade cost level. The quote between the two welfare levels is:

$$\frac{W_{t=0}^{B}}{W_{t=t}^{B}} = \frac{2 (3 - 2\theta)(2 - \theta^2)^2}{(3 - 2\theta^2)(4 - 3\theta^2 + \theta^3)} > 1.$$  

We can show that $W$ is continuous and strictly convex for trade costs between free trade and the prohibitive level. It follows that welfare is U-shaped. To establish iii) note that the quote between the lowest welfare level with trade and the welfare level in autarchy is

$$\frac{W_{t=t_{\min}}^{B}}{W^{M}} = \frac{4 (9 - 4\theta^2)}{12 + 2\theta^4 - 9\theta^2} > 1,$$

which shows that welfare under autarchy is strictly lower than welfare under trade for any $t \in [0, \hat{t}]$.

### 2.2 Cournot vs. Bertrand

As we just established, the local welfare result from Brander-Krugman carries over to the case of a Bertrand duopoly with differentiated goods, while the global result does not. In particular, the analysis shows that trade in a Bertrand duopoly can not reduce welfare compared to autarchy. Since the positive effect of trade is due to a combination of competition and consumers’ love-of-variety, this raises the question whether love-of-variety may possibly reverse the global welfare result in the Cournot model as well. This section accordingly address the following question: Does trade always increase welfare compared to autarchy, if products are sufficiently differentiated?

For this purpose, we contrast welfare in the Bertrand model with welfare in the Cournot model. The reaction curves assuming Cournot competition (but keeping all the other assumptions above) are

$$q_{H}^{C2}(q_{F}) = \frac{a - \theta b q_{F} - c}{2b},$$

$$q_{F}^{C2}(q_{H}) = \frac{a - \theta b q_{H} - c - t}{2b}.$$  

\[6\]
Solving for the equilibrium quantities we can calculate the prohibitive trade cost \( q_C^F = 0 \) as

\[
\tau^C_2 = \frac{(a - c)(2 - \theta)}{2},
\]

which corresponds to point A ("autarchy") in Figure 1 with Bertrand competition. Contrary to the Bertrand model, we find that in the Cournot model trade can reduce welfare compared to autarchy, for any level of product differentiation.

Proposition 2 Let utility be given by Eq (1) with \( \theta \in (0, 1] \). Let firms have constant marginal costs and per unit transport cost. Assume that there is one firm from each country and that these firms compete à la Cournot. Then the minimum welfare level with trade is lower than in autarchy, for any degree of product differentiation.

Proof. Using the equilibrium quantities (7) and (8) we can calculate welfare under trade and compare with autarchy to establish that

\[
\frac{W^{|C^2|}_{\tau = \tau_{\text{min}}}}{W^M} = \frac{4}{3} \left( \frac{9 - \theta^2}{12 - \theta^2} \right),
\]

which is strictly less than unity for any \( \theta \in (0, 1] \).

This is a striking result, to the best of our knowledge new and at first counterintuitive.\(^5\) One would have thought that if there is sufficient product differentiation trade is always welfare improving. As shown that intuition is not correct - trade can lower welfare as long as demands for products are not completely independent.\(^6\) The intuition for the present result is nevertheless quite straightforward. At the prohibitive trade cost there is no trade with foreign varieties. A slight reduction of the transport cost would lead the foreign firm to export. This has three effects on welfare. The utility of consumers in the domestic market is marginally increased by the imports; the price-adjusted utility of imported goods is marginally higher than the price-adjusted utility of domestic products. In addition, trade generates a profit for foreign firms. The margin for traded goods is positive but very close to zero. The price barely covers the cost when the trade barrier is close to the prohibitive level. Finally, the substitution

\(^5\) The closest precursor to the analysis in this section is perhaps Bernhofen (2001) who examines Cournot competition in differentiated goods, but focuses on other issues.

\(^6\) The intuition is correct in the sense though that the more differentiated that the two products are, the smaller is the difference between the autarchy welfare and lowest possible welfare under trade.
away from domestic goods to imported varieties has a negative effect on profits. Close to a prohibitive transport cost the margin for domestic goods is strictly positive. Substitution from domestic products to imports consequently results in a non-marginal reduction of domestic profits. To sum up, an increased trade volume reduces welfare close to the prohibitive trade barrier since the only first-order effect is the reduced domestic profit.

Figure 2 about here

Figure 2 illustrates how welfare depends on trade costs for the Cournot and Bertrand case, respectively. It is obvious that the fundamental difference between the Cournot and the Bertrand duopoly model is not due to the welfare level in autarchy. Instead, the significant difference is due to the strategic nature of Bertrand and Cournot competition. Moving from monopoly to duopoly at the prohibitive trade cost in a Cournot model is a small step. Prices change only marginally and quantities are essentially unchanged. Moving from unconstrained monopoly to a duopoly with positive trade flows in a Bertrand model, on the other hand, is a discrete change. The equilibrium with trade is significantly more competitive than unconstrained monopoly.

3 Competition in autarchy

The market structure analyzed so far is rather special. The domestic producer has a monopoly in autarchy and the outcome without trade, accordingly, does not depend on competition between producers. It could be argued that a more typical situation is one where there is at least some competition, also in autarchy. We extend the model in the previous section and let there be four firms, two domestic producers (1 and 2) and two foreign producers (3 and 4). Markets are symmetric and we again focus on the home market. A representative consumer in the home market has the following utility function

\[
U(q) = \sum_{i=1,\ldots,4} \left( aq_i - \frac{1}{2} b q_i^2 \right) - \theta b (q_1 q_2 + q_1 q_3 + q_1 q_4 + q_2 q_3 + q_2 q_4 + q_3 q_4) + m
\] (11)
where $q_i$ is the quantity of firm i’s variety and $m$ is the utility of money. Utility maximization subject to the budget constraint gives the following inverse demand functions:

$$ p_i = a - bq_i - \theta b \sum_{j \neq i} q_j. $$

Re-arrange these equations to obtain the demand functions:

$$ q_i = \frac{(1 - \theta) a - (1 + 2\theta) p_i + \theta \sum_{j \neq i} p_j}{(1 + 3\theta)(1 - \theta) b} $$

and the system of demand functions is linear in all prices. Firms maximize profits and first order conditions can be used to derive reaction curves in the trade equilibrium (when all four firms have positive sales in market H). The reaction curve for a domestic firm (1 and 2) is

$$ p_i = \frac{(1 - \theta) a + \theta \sum_{j \neq i} p_j}{2(2\theta + 1)} + \frac{c}{2} $$

and for a foreign firm (3 and 4)

$$ p_i = \frac{(1 - \theta) a + \theta \sum_{j \neq i} p_j}{2(2\theta + 1)} + \frac{c}{2} + \frac{t}{2}. $$

In Appendix B we solve for the equilibrium prices and quantities. These equilibrium quantities can be inserted in the welfare function to find the welfare level for non-prohibitive trade costs.

Welfare is illustrated in Figure 3. In the figure, $\hat{t}$ denotes the trade costs at which it is profitable for one foreign producer to enter and $\hat{t}$ the trade costs at which both foreign firms will export. The figure shows that, for any given trade cost, welfare is lower if products are closer substitutes. The reason for this is that the value of product variety is low if the products are close substitutes. The figure also shows that imports occur for a much wider range of trade costs if products are distant substitutes. Competition between distant varieties is less aggressive and leaves more room for inefficient producers (in this case more costly imports). In addition, product variety also has an additional value for consumers and the

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For ease of comparison we think of the game as a two-stage game such that welfare equals the autarchy welfare until the point where trade actually occurs. This is not important for any conclusions that we draw.
price is consequently higher and, therefore, stimulates more imports.

Figure 3 about here

Welfare is a continuous function in $t$ (for $t < \hat{t}$) and the first order condition, $W'_t = 0$, gives the minimum welfare level. The solution to the first order condition is unique and, in the interval of non-prohibitive trade costs, the welfare has its minimum at

$$t_{\text{min}}^{B4} = \frac{3(a - c)(1 - \theta)(5\theta + 2)^2}{12\theta^4 + 19\theta^3 + 91\theta^2 + 64\theta + 12},$$

(16)

which is a trade equilibrium ($0 \leq t_{\text{min}}^B \leq \hat{t}^B$). Welfare at $t_{\text{min}}^B$ is

$$W|_{t_{\text{min}}^B}^{B4} = \frac{3(1 + 2\theta)(8\theta + 3)(a - c)^2}{b(12\theta^4 + 19\theta^3 + 91\theta^2 + 64\theta + 12)},$$

(17)

which is declining in product homogeneity ($\theta$).

The lowest welfare level with trade can be compared with the welfare level in autarchy. The equilibrium autarchy-prices in the domestic market are

$$\hat{p}_1^{B2} = \hat{p}_2^{B2} = \frac{a(1 - \theta) + c}{2 - \theta},$$

(18)

and corresponding quantities are

$$\hat{q}_1^{B2} = \hat{q}_2^{B2} = \frac{a - c}{(\theta + 1)(2 - \theta)b},$$

(19)

which can be inserted in the welfare function

$$W = a(q_1 + q_2) - \frac{1}{2}b(q_1^2 + q_2^2 + 2\theta q_1 q_2) - c(q_1 + q_2),$$

(20)

to obtain the Bertrand welfare level in autarchy

$$W^{B2} = W|_{t > \hat{t}} = \frac{(3 - 2\theta)(a - c)^2}{(2 - \theta)^2(\theta + 1)b},$$

(21)
Now, the welfare level in autarchy can be compared with the lowest welfare level with trade, \( W|_{t=\min} \) by taking the quote between welfare levels, i.e.

\[
\frac{W_B^{t_{\min}}}{W_B^{t_2}} = \frac{3(\theta + 1)(1 + 2\theta)(8\theta + 3)(2 - \theta)^2}{(12\theta^4 + 19\theta^3 + 91\theta^2 + 64\theta + 12)(3 - 2\theta)}
\]  

(22)

which is continuous in \( \theta \). If products are independent or homogenous, the quote is equal to unity and welfare levels are equal. The quote has its maximum at \( \theta = 0.4 \), where welfare with trade is higher than welfare in autarchy. The quote has a minimum at \( \theta = 0.9 \), where minimum welfare with trade is lower than welfare in autarchy. Welfare in autarchy is higher than the minimum welfare with trade, if products are sufficiently close substitutes.

**Proposition 3** Let utility be given by Eq (11) with \( \theta \in (0,1) \). Let firms have constant marginal costs and per unit transport cost. Assume that there is a Bertrand duopoly in each country. Then the lowest welfare level with trade is lower than welfare under autarchy, if goods are sufficiently close substitutes \( (\theta \in (0.76,1)) \).

**Proof.** Set the quote in Eq (22) quote to 1 and solve for critical \( \theta \), which is equivalent to

\[
\theta^2 \left( 20 - 100\theta^2 + 8\theta + 72\theta^3 \right) = 0
\]

where \( \theta_1 = \theta_2 = 0 \) are two obvious roots and \( \theta_3 = 1 \) is a third root. The two remaining roots are

\[
\theta = \frac{7}{36} + \frac{1}{36}\sqrt{409} = 0.75622
\]

and

\[
\theta = \frac{7}{36} - \frac{1}{36}\sqrt{409} = -0.36733
\]

The only root in the interior \((0,1)\) is \( \theta = 0.75622 \). At this point the quote is decreasing. The derivative is:

\[
\frac{d}{d\theta} \left( \frac{W_{\min}}{W_D} \right) = -0.06 < 0
\]
and trade can reduce welfare compared to autarchy for

$$\theta > \frac{7}{36} + \frac{1}{36}\sqrt{409}.$$ 

Thus $W_{t=t_{\text{min}}}^{B_4} < W^{B_2}$ if $1 > \theta > \frac{7}{36} + \frac{1}{36}\sqrt{409} = 0.75622$. ■

The analysis in this section illustrates the importance of competition in autarchy for the welfare effects of trade in a Bertrand model. While more than one domestic producer in autarchy and relatively similar products, are two necessary conditions for reciprocal dumping to possibly generate negative welfare results in a Bertrand model, neither product differentiation nor monopoly in autarchy affects this conclusion in the Cournot model. The difference between the Bertrand model and the Cournot model is shown in Figure 4.

Figure 4 about here

4 Conclusion

This paper shows that trade can reduce welfare in a model with price competition and differentiated products under standard assumptions. If products are sufficiently close substitutes and the autarchy equilibrium is sufficiently competitive, welfare is higher in autarchy than the lowest level with trade. Adding more products that are close substitutes to a market with price competition does not add much consumer value and prices are already close to marginal cost due to the aggressive nature of price competition. The waste of real resources in trade, consequently, dominates the effects of competition as well as market expansion for intermediate and high trade costs.

We focus on the possibility of trade having a negative effect on welfare. This might seem a bit odd as really the main message from our paper is that trade is almost always welfare increasing. The question we are interested in is if the qualification "almost" encompasses a significant range of parameter values and modeling choices or not. Indeed, the details of the results will be sensitive to how we model utility.\(^8\) On a related note the welfare effects of a greater choice of products has been a lively theme

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\(^8\) We have used a form of utility where the market expands as more products become available (the Bowley form). Another form of utility that is sometimes used to generate simple linear demand functions for differentiated goods is the one associated with Shubik and Levitan where the market does not expand as more products are added (see Shubik and
in the empirical literature (see for instance Petrin, 2002 or Ackerberg and Rysman, 2005). Indeed, while theoretical work is important to understand the robustness of these welfare effects we believe that empirical work will be crucial for resolving these issues. As a start, in Friberg and Ganslandt (2005) we examine the welfare effects of moving to autarchy for the Swedish bottled water market and do not find that trade lowers welfare on that market. While one study is not enough to put the concerns raised by the above analysis to rest, one may hope that additional empirical studies will examine the relevance of the proposition that trade may lower welfare so that we get some robust findings as to whether we should worry about this aspect of trade.

References


Levitan, 1980, ch 6 or Martin 2002, for a discussion). Imports do then not expand the market but only replace domestic products. We conjecture that there would be a greater range of transport costs for which trade had a negative association with welfare than under the present assumptions.


Appendix A

This appendix details a number omitted equations from the analysis in section 2 and 3. A full appendix of calculations are available for downloads at http://www.hhs.se/personal/friberg. The inverse demand functions for variety H and F are given by:

\[ p_H = a - bq_H - \theta bq_F, \]  
\[ p_F = a - bq_F - \theta bq_H. \]  

(23) 

(24)

Re-arrange these equations to obtain the demand functions for H and F:

\[ q_H = \frac{a(1-\theta)}{b(1-\theta^2)} - \frac{1}{b(1-\theta^2)} p_H + \frac{\theta}{b(1-\theta^2)} p_F \]  
\[ q_F = \frac{a(1-\theta)}{b(1-\theta^2)} + \frac{\theta}{b(1-\theta^2)} p_H - \frac{1}{b(1-\theta^2)} p_F. \]  

(25) 

(26)

The two first order conditions give the unique Bertrand equilibrium (assuming that the import volume is positive):

\[ p_{\text{B}H} = \frac{2a - a\theta - a\theta^2 + 2c + c\theta + t\theta}{4 - \theta^2} \]  
\[ p_{\text{B}F} = \frac{2a - a\theta - a\theta^2 + 2c + c\theta + 2t}{4 - \theta^2}. \]  

(27) 

(28)

Insert the equilibrium prices in (25) and (26) to obtain the equilibrium quantities, denoted \( q_{\text{B}H} \) and \( q_{\text{B}F} \). In order to find the highest trade cost at which there are imports we set \( q_{\text{B}F} = 0 \) and solve for the critical threshold. The prohibitive trade cost is

\[ \hat{t}^B = \frac{(a - c)(2 - \theta - \theta^2)}{2 - \theta^2}. \]  

(29)
At this trade cost the unique Bertrand equilibrium is point B in Figure 1.

We first evaluate welfare in the interior Bertrand equilibrium with trade (points on the line between F and B). For an interior equilibrium with trade, total welfare in the home market is

$$W^B = U(q^B_H, q^B_F) + \pi_H (p^B_H, p^B_F) + \pi_F (p^B_H, p^B_F). \tag{30}$$

We are now in a position to calculate the welfare at different levels of transport costs. Using the equilibrium values of prices and quantities in Equation (30) we express equilibrium welfare as a function of parameter values. The resulting expression is long and unwieldy. However, we may take advantage of the fact that we are interested in evaluating it at different levels of transport costs. Using particular values for the transport costs the welfare function simplifies to quite manageable expressions. Welfare with costless trade, i.e. $t = 0$ (point F), is

$$W^B_{t=0} = \frac{2ac - 2a^2}{b\theta^2 - b\theta - 2b} + \frac{2ac - 2c^2}{b\theta^2 - b\theta - 2b} + \frac{2ac - a^2 - c^2}{4b - 3b\theta^2 + b\theta^3}. \tag{31}$$

while welfare at the prohibitive level, i.e. $t = \hat{t}$ (point B), is

$$W^B_{t=\hat{t}} = \frac{ac - a^2}{b\hat{\theta}^2 - 2b} + \frac{ac - c^2}{b\hat{\theta}^2 - 2b} + \frac{2ac - a^2 - c^2}{8b - 8b\hat{\theta}^2 + 2b\hat{\theta}^4}. \tag{32}$$

In the interval of non-prohibitive trade costs the welfare has its minimum at

$$t_{\text{min}}^B = \left( \frac{(1 - \theta) (3 - 2\theta) (a - c) (2 + \theta)^2}{2\theta^4 - 9\theta^2 + 12} \right). \tag{33}$$

and at this trade cost, welfare is

$$W^B_{t=t_{\text{min}}} = \frac{(9 - 4\theta^2) (a - c)^2}{2b (2\theta^4 - 9\theta^2 + 12)}. \tag{34}$$

which is the lowest welfare level with trade.

We proceed with the analysis for trade costs at, and above, the trade-deterring level. Assume that
the foreign firm takes sequential decisions (a simultaneous game is analyzed next). The foreign firm first takes a decision to enter or stay out of market H and then, subject to entry, sets a profit maximizing price in a non-cooperative fashion. The foreign firm enters market H if the expected profit in the Nash equilibrium is non-negative. Accordingly, in the sequential game the foreign firm enters if \( t \leq \hat{t} \) and stays out if \( t > \hat{t} \).

For trade costs above the critical level (\( t > \hat{t} \)), the home firm maximizes the monopoly profit

\[
\pi_H = \left( \left( \frac{a}{b} - \frac{p_H}{b} \right)(p_H - c) \right)
\]

and the unique equilibrium is

\[
p_H^* = \frac{a + c}{2}.
\]

and welfare is consequently

\[
W^M = W |_{t > \hat{t}} = \frac{3(a-c)^2}{8b}.
\]

In the simultaneous game the foreign firm takes a decision to enter or stay out of market H and simultaneously sets a profit maximizing price in a non-cooperative fashion. Accordingly, in the simultaneous game the home firm must set a price such that it is unprofitable for the foreign firm to deviate from a price that yields no imports. The reaction function of the foreign firm is

\[
p_F(p_H) = \frac{1}{2}a(1-\theta) + t + \theta p_H
\]

and we insert this in the demand function for the foreign firm and solve for prices such that no importation occurs, i.e. \( q_F = 0 \), which yields

\[
p_H = \frac{c + t - a(1-\theta)}{\theta}
\]

\[
p_F = c + t
\]
and it is obvious that the foreign firm makes no profit in the home market at these prices. The home
firm’s price is an increasing function in \( t \). This price is above the monopoly level for very high trade
costs. Consequently, the home firm sets the autarchy (monopoly) price for sufficiently high trade costs,
i.e. trade costs above the threshold

\[
\tilde{t} = \frac{(a - c) (2 - \theta)}{2}.
\]  

(41)

In the intermediate interval, i.e. \( \hat{t} < t < \tilde{t} \), welfare is

\[
W|_{\hat{t}<t<\tilde{t}} = \frac{(a - c - t) (2a\theta - a + c + t - 2c\theta)}{b\theta^2}.
\]  

(42)

and welfare is a declining function in the trade cost, \( t \). In the simultaneous game, the welfare function
is continuous at \( t = \hat{t} \) and the function is strictly decreasing to the autarchy level at \( \tilde{t} \).

We not turn to the Cournot game that we examine in section 3. The unique Cournot equilibrium is

\[
q_{C2}^H = \frac{a - c}{(2 + \theta) b} + \frac{\theta t}{(4 - \theta^2) b},
\]  

(43)

\[
q_{C2}^F = \frac{a - c}{(2 + \theta) b} - \frac{2t}{(4 - \theta^2) b}.
\]  

(44)

The prohibitive trade cost \( (q_{C2}^F = 0) \) is

\[
\tilde{t}_{C2} = \frac{(a - c) (2 - \theta)}{2},
\]  

(45)

which corresponds to point A in Figure 1 with Bertrand competition. Total welfare for an interior
equilibrium with trade in the Cournot model

\[
W_{C2} = U(q_{C2}^H, q_{C2}^F) + \pi_H (p_{C2}^H, p_{C2}^F) + \pi_F (p_{C2}^H, p_{C2}^F).
\]  

(46)

which is continuous and differentiable in \( t \). We set \( W' = 0 \) and solve for \( t \) to obtain the minimum, which
is at the interior trade cost

\[
t_{C2}^{min} = \frac{(a - c) (3 + \theta) (2 - \theta)^2}{12 - \theta^2}.
\]  

(47)
Appendix B

This appendix gives the omitted details from the analysis of the 4 firm case. We commence with Bertrand competition. Solving the first order conditions simultaneously yield the unique Bertrand equilibrium (assuming that the import volume is positive). The equilibrium price for a domestic producer (i=1 and 2) is

\[ p_i^{B4} = \left( \frac{1 - \theta}{2 + \theta} \right) a + \left( \frac{1 + 2\theta}{2 + \theta} \right) \left( c + \frac{2\theta t}{5\theta + 2} \right), \quad (48) \]

and for a foreign firm (j=3 and 4)

\[ p_j^{B4} = \left( \frac{1 - \theta}{2 + \theta} \right) a + \left( \frac{1 + 2\theta}{2 + \theta} \right) \left( c + \frac{(3\theta + 2)t}{5\theta + 2} \right), \quad (49) \]

and we insert the equilibrium prices in (13), to obtain the equilibrium quantities for a domestic producer

(1 and 2)

\[ q_i^{B4} = \left[ \frac{2\theta + 1}{(2 + \theta)(3\theta + 1)} \right] \frac{a - c}{b} + \left[ \frac{2\theta (2\theta + 1)^2}{(5\theta + 2)(2 + \theta)(3\theta + 1)(1 - \theta)} \right] \frac{t}{b}, \quad (50) \]

and for a foreign firm (3 and 4)

\[ q_j^{B4} = \left[ \frac{2\theta + 1}{(2 + \theta)(3\theta + 1)} \right] \frac{a - c}{b} - \left[ \frac{(2\theta + 1)(2 + 5\theta - \theta^2)}{(5\theta + 2)(2 + \theta)(3\theta + 1)(1 - \theta)} \right] \frac{t}{b}. \quad (51) \]

The critical trade cost when one foreign firms finds it unprofitable to be active in market H is implicitly given by \( q_j^B(t) = 0 \) for \( j = 3, 4 \).

\[ \hat{\tau}^B = \hat{\tau}^{B4} = \frac{(a - c)(5\theta + 2)(1 - \theta)}{2 + 5\theta - \theta^2}. \quad (52) \]

With only one foreign firm active in market H, the market is a triopoly (two domestic and one foreign
The unique triopoly equilibrium is

\[
q_1 = q_2 = \left[ \frac{1 + \theta}{1 + 2\theta} \right] \frac{a - c}{2b} + \left[ \frac{\theta (1 + \theta)^2}{2(3\theta + 2)(1 - \theta)(1 + 2\theta)} \right] \frac{t}{b}, \tag{53}
\]

\[
q_3 = q_4 = \left[ \frac{1 + \theta}{1 + 2\theta} \right] \frac{a - c}{2b} + \left[ \frac{(1 + \theta)(\theta^2 - 3\theta - 2)}{2(3\theta + 2)(1 - \theta)(1 + 2\theta)} \right] \frac{t}{b}. \tag{54}
\]

Eventually, for a sufficiently high trade cost, it is unprofitable for any foreign firms to export to market H and the prohibitive trade cost is:

\[
\hat{t}^B = \hat{t}^B_3 = \frac{(a - c)(3\theta + 2)(1 - \theta)}{2 + 3\theta - \theta^2}. \tag{55}
\]

Let us now proceed to the analysis if Cournot competition. The unique Cournot equilibrium is

\[
q_1^{C_4} = q_2^{C_4} = \frac{a - c}{b(3\theta + 2)} + \frac{2\theta t}{b(3\theta + 2)(2 - \theta)}, \tag{56}
\]

\[
q_3^{C_4} = q_4^{C_4} = \frac{a - c}{b(3\theta + 2)} - \frac{(2 + \theta) t}{b(3\theta + 2)(2 - \theta)}. \tag{57}
\]

The prohibitive trade cost \( q_3^{C_4} = q_4^{C_4} = 0 \) is

\[
\hat{t}^{C_4} = \frac{(2 - \theta)(a - c)}{(2 + \theta)}. \tag{58}
\]

Proceeding to the welfare analysis we compute total welfare for an interior equilibrium with trade in the Cournot model

\[
W^{C_4} = U(q_{H_1}^{C_4}, q_{F_1}^{C_4}) + \pi_H (p_{H_1}^{C_4}, p_{F_1}^{C_4}) + \pi_F (p_{H_1}^{C_4}, p_{F_1}^{C_4}) \tag{59}
\]

which is continuous and differentiable in \( t \). We set \( W' = 0 \) and solve for \( t \) to obtain the minimum, which is at the interior trade cost

\[
t_{C_4}^{t_{\text{min}}} = \frac{3(\theta + 1)(a - c)(2 - \theta)^2}{12 + 16\theta + 3\theta^2 - 3\theta^3}. \tag{60}
\]

Evaluating welfare at this point we find that the minimum welfare level with trade is

\[
W^{C_4}_{t_{\text{min}}} = \frac{3(\theta + 1)(3 - \theta)(a - c)^2}{b(12 + 16\theta + 3\theta^2 - 3\theta^3)}. \tag{61}
\]
There is a significant difference between the Cournot equilibrium and the Bertrand equilibrium when two domestic firms are active. The Bertrand equilibrium is more competitive as firms compete in prices. The unique Cournot equilibrium, on the other hand, is

\begin{align*}
\hat{q}_1^C &= \hat{q}_2^C = \frac{a-c}{b(2+\theta)} \\
\hat{q}_3^C &= \hat{q}_4^C = 0
\end{align*}

and welfare is

\begin{equation}
W^{C2} = W^{t\geq\hat{t}} = \frac{(3 + \theta)(a-c)^2}{b(\theta+2)^2}
\end{equation}
Figure 1. Reaction curves in a Bertrand duopoly
Figure 2. Welfare in a Bertrand and Cournot duopoly as a function of transport costs (a=2, b=1, c=0.5, $\theta = 0.9$)
Figure 3. Welfare in a Bertrand oligopoly (a=2, b=1, c=0.5).
Figure 4. Welfare in a Bertrand and Cournot oligopoly (a=2, b=1, c=0.5)